Effects of extra space-time dimensions on the Fermi constant

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Effects of Kaluza-Klein excitations associated with extra dimensions with large radius compactifications on the Fermi constant are explored. It is shown that the current precision determinations of the Fermi constant, of the fine structure constant, and of the W and Z mass put stringent constraints on the compactification radius. The analysis excludes one extra space-time dimension below ~ 1.6 TeV, and excludes 2, 3, and 4 extra space dimensions opening simultaneously below ~ 3.5 TeV, 5.7 TeV, and 7.8 TeV at the 90% C.L. The implications of these results for future collider experiments are discussed. [S0556-2821(99)05821-X]

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Kaluza-Klein theories have a long and rich history [1,2]. The TeV scale strings provide a new impetus for studying these theories in the context of low energy phenomenology. Until recently much of string phenomenology has been conducted in the framework of the weakly coupled heterotic string where a rigid relationship exists between the string scale ($M_{\rm str}$) and the Planck scale $M_{\rm Planck}$ [3], i.e., $M_{\rm str} \sim g_{\rm str} M_{\rm Planck}$ where $M_{\rm Planck} = (8\pi G_N)^{-1/2}$, and where G_N is Newton's constant and numerically $M_{\rm Planck} = 1.2 \times 10^{19}$ GeV. However, recently the advent of string dualities has opened the new possibility which relates the strong coupling regime of one string theory to the weak coupling limit of another. Thus it is conjectured that the strongly coupled SO(32) heterotic string compactified to four dimensions is equivalent to a weakly coupled type I string compactified on four dimensions. In this context the string scale can be very different [4,5]. While one is very far from generating realistic TeV string models the generic features of models of this type with low values of the string scales can nonetheless be studied [6–8]. Here we adopt the picture that matter resides in $D = d + 4$ dimensions while gravity propagates in the 10 dimensional bulk. In the context of type I strings one may conjecture that matter resides on a p-brane ($p = d + 3$) with compactification of d dimensions occurring internal to the brane while the compactification of the remaining $(6 - d)$ dimensions occurs in directions transverse to the p-brane. We focus on the d compactifications internal to the brane and the compactifications transverse to the brane will not concern us here. We shall work within the framework of an effective field theory, which is a valid approximation in the domain of energy investigated below. The main motivation of this paper is to analyze the effects of the Kaluza-Klein excitations associated with the extra dimensions on the Fermi constant G_F which is one of the most accurately determined quantities in particle physics [9]. In our analysis we shall assume that a number d of extra dimensions open at a low scale—each associated with a common radius of compactification $R = M_R^{-1}$.

We consider the 5D case first. Our starting point is the minimal supersymmetric standard model (MSSM) Lagrangian in 5D and we exhibit here a few terms to define notation

$$L_5 = -\frac{1}{4} F_{MN} F^{MN} - (D_M H_i)^\dagger (D^M H_i) - \bar{\psi} \frac{1}{i} \Gamma^M D_M \psi - V(H_i) + \cdots, \tag{1}$$

where M, N are the five dimensional indices that run over values 0,1,2,3,5, H_i ($i = 1,2$) stand for two Higgs hypermultiplets which contain the MSSM Higgs boson, $D_M = \partial_M - i g^{(5)} A_M$, where $g^{(5)}$ are the gauge coupling constants and the SU(3)×SU(2)×U(1) indices are suppressed, and $V(H_i)$ is the Higgs potential. This theory is constructed to have $N = 1$ supersymmetry in 5D. We compactify this theory on S^1/Z_2 with the radius of compactification R. In our analysis we assume that the gauge fields and the Higgs fields live in the five dimensional bulk while the quarks and leptons are confined to a four dimensional wall, i.e, at a Z_2 fixed point [10]. After compactification the resulting spectrum contains massless modes with $N = 1$ supersymmetry in 4D, which precisely form the spectrum of MSSM in 4D, and massive Kaluza-Klein modes which form $N = 2$ multiplets in 4D. The modes in $4D$ can be further labeled as even or odd under the action of Z_2. The MSSM spectrum and their Kaluza-Klein excitations are even under Z_2 while the remaining $N = 1$ massive set of Kaluza-Klein fields are odd. We carry out a spontaneous breaking of the electroweak symmetry in 5D which gives electroweak masses to the W and Z gauge bosons and to their Kaluza-Klein modes in addition to the compactification masses, i.e., Kaluza-Klein modes have masses $(m_i^2 + n^2 M_R^2)$, $n = 1,2,3, \ldots, \infty$, where m_i^2 arise from electroweak symmetry breaking and $n^2 R^2$ arise from compactification of the fifth dimension. In 4D a rescaling of coupling constants is needed, i.e., $g_i^{(5)}/\sqrt{\pi R} = g_i$ where g_i are the gauge coupling constants in four dimensions. The interactions of the fermions to zero modes and to Kaluza-Klein modes after rescaling is given by

$$L_{\rm int} = g_i j^\mu \left(A_{\mu i} + \sqrt{2} \sum_{n=1}^{\infty} A_{\mu i}^n \right), \tag{2}$$

where $A_{\mu i}$ are the zero modes and $A_{\mu i}^n$ are the Kaluza-Klein modes.

The Fermi constant is very accurately known from the weak interaction processes. Its current experimental value obtained from the muon lifetime including the complete two-loop quantum electromagnetic contributions is [9]

$$G_F = 1.16637(1)\times 10^{-5}\ \text{GeV}^{-2} \tag{3}$$

where the number in the parenthesis is the error. We compare now the experimental result on G_F of Eq. (3) with the predictions of G_F of Eq. (3) in the standard model (SM). In the on-shell scheme $G_F^{\rm SM}$ is given by [11]

$$G_F^{\rm SM} = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2\theta_W(1-\Delta r)} \tag{4}$$

where $\sin^2\theta_W = (1 - M_W^2/M_Z^2)$ in the on-shell scheme. The fine structure constant α (at $Q^2=0$) is very accurately known, i.e., $\alpha^{-1} = 137.0359$, and Δr is the radiative correction and is determined to be [12] $\Delta r = 0.0349 \mp 0.0019 \pm 0.0007$ where the first error comes from the error in the mass of the top quark ($m_t = 175\pm 5$ GeV) and the second error comes from the uncertainty of $\alpha(M_Z)$. With the above one can use the measured values of M_W and M_Z to derive the value of $G_F^{\rm (SM)}$. The currently measured values of M_W and M_Z are $M_W = 80.39\pm 0.06$ GeV [13] and $M_Z = 91.1867 \pm 0.0020$ GeV [12]. Using the above determinations one finds that the standard model prediction of G_F is in excellent accord with experiment. Thus we conclude that the contribution of the Kaluza-Klein modes must fall in the error corridors of the standard model to be consistent with current experiment. Since the fine structure constant and M_Z are very accurately known most of the error in the evaluation of $G_F^{\rm SM}$ arises from the errors in the measurement of the W mass and in the evaluation of the radiative correction Δr. To exhibit the relative contribution of the errors from these sources to G_F we find that

$$\Delta G_F/G_F|_{\rm SM} \simeq \sqrt{4(1/\sin^2\theta_W - 2)^2(\delta M_W/M_W)^2 + (\delta\Delta r)^2}. \tag{5}$$

The quantity $2(1/\sin^2\theta_W - 2)\sim 5$ is accidentally large and thus the error in M_W dominates the error in the standard model contribution to the Fermi constant. Using the above analysis we find that $G_F^{\rm SM} = (1.16775\pm 0.0049)\times 10^{-5}\ \text{GeV}^{-2}$. We note that while the error in the measurement of the W mass is less than 0.1% it gets enlarged to around 0.5% due to the enhancement factor of 5. Even so the error corridor of $G_F^{\rm SM}$ is very narrow and places a strong constraint on new physics. Thus we use this corridor to constrain the Kaluza-Klein contributions to the Fermi constant. In our analysis we shall assume that the radiative corrections from the Kaluza-Klein states to Δr are small. This assumption will turn out to be *a posteriori* justified in view of the largeness of the limits on masses of the Kaluza-Klein (KK) excitations obtained in this analysis. Under the above assumption we then require that $G_F^{\rm KK}$ be limited by the error in $\Delta G_F^{\rm SM}$, i.e.,

$$\Delta G_F^{\rm KK}/G_F^{\rm SM} \le \pm 0.82\times 10^{-2} \quad (90\%\ \text{C.L.}). \tag{6}$$

For the case of one extra dimension, after compactification and integration over the W boson and its Kaluza-Klein excitations we obtain the effective Fermi constant to leading order in M_W/M_R to be

$$G_F^{\rm eff} \simeq G_F^{\rm SM}\left(1 + \frac{\pi^2}{3}\frac{M_W^2}{M_R^2}\right). \tag{7}$$

Defining $\Delta G_F^{\rm KK} = G_F^{\rm eff} - G_F^{\rm SM}$, identifying $G_F^{\rm eff}$ with the experimental value of the Fermi constant, and using Eq. (6) we find $M_R > 1.6$ TeV (90% C.L.). This limit on M_R is stronger than for the case when one has just an extra W recurrence. Thus if one had an extra W recurrence with a mass $M_{W'}$, the analysis above will give a limit $M_{W'} > 905$ GeV. The stronger limit for the Kaluza-Klein case is due to an enhancement factor of $\pi/\sqrt{3}$. This factor arises in part due to summation over the tower of Kaluza-Klein states and in part due to the coupling of the 4D fermions to the Kaluza-Klein gauge bosons being stronger by a factor of $\sqrt{2}$ relative to the couplings of the fermions to the zero mode gauge bosons as can be seen from Eq. (2). The current experimental limit on the recurrence of a W is $M_{W'} > 720$ GeV given by the analysis at the Fermilab Tevatron $p\bar{p}$ collider data with the DØ detector [14]. Thus our result on M_R is much stronger than the current experimental limit on $M_{W'}$.

Another independent constraint on M_R can be obtained from atomic parity experiments. The atomic parity violations arise from the Z exchange and the low energy effective interaction governing the violation is [15] $L_{PV}^{\rm SM} = (G_F^{\rm SM}/\sqrt{2})\Sigma_i C_{1i}\bar{e}\gamma_\mu\gamma_5 e\bar{q}_i\gamma^\mu q_i$. In SM the measured quantity is $Q_W^{\rm SM} = 2[(2Z+N)C_{1u} + (Z+2N)C_{1d}]$ where Z is the number of protons and N is the number of neutron in the atomic nucleus being considered. The exchange of the Kaluza-Klein Z bosons contribute an additional interaction, i.e.,

$$L_{PV}^{\rm KK} = (\Delta G_F^{\rm KK}/\sqrt{2})\sum_i C_{1i}\bar{e}\gamma_\mu\gamma_5 e\bar{q}_i\gamma^\mu q_i. \tag{8}$$

The most accurate atomic parity experiment is for cesium which gives

$$Q_W^{\rm exp}({\rm Cs}) = -72.41\pm 0.25\pm 0.80 \tag{9}$$

while the standard model gives

$$Q_W^{\rm SM}({\rm Cs}) = -73.12\pm 0.06. \tag{10}$$

The above result gives $\Delta Q_W^{\rm exp} - Q_M^{\rm SM} = 0.71\pm 0.84$ where we have added the errors in quadrature. We define the Kaluza-Klein contribution to Q_W by $\Delta Q_W^{\rm KK} = (\Delta G_F^{\rm KK}/G_F^{\rm SM})Q_W^{\rm (SM)}$ and require that $\Delta Q_W^{\rm KK}$ be limited by ΔQ_W, i.e., $\Delta Q_W^{\rm KK} \le \Delta Q_W$. This leads to a constraint on M_R of $M_R > 1.4$ TeV (90% C.L.) which is less strong than the limit obtained from the analysis of G_F but still quite impressive.

The current accuracy of the experimental determinations of other electroweak quantities produce less stringent constraints on M_R.

The above analysis can be extended to d extra dimensions. We consider here only compacifications with common compactification radius R and Z_2 type orbifolding. Integration on the W boson and its Kaluza-Klein excitations similar to the 5D case gives the following result for the sum of the standard model and Kaluza-Klein mode contributions

$$G_F^{\mathrm{eff}} = G_F^{\mathrm{SM}} K_d\left(\frac{M_W^2}{M_R^2}\right) \tag{11}$$

where K_d is the Kaluza-Klein dressing of the Fermi constant

$$K_d(c) = 1 + \sum_{p=1}^{d} (2^p \ {}^dC_p) C_p(c). \tag{12}$$

Here ${}^dC_p = d!/p!(d-p)!$, $C_p(c) = \Sigma_{\vec{k}_p}[c/(c+\vec{k}_p^2)]$ where $\vec{k}_p = (k_1, k_2, \dots, k_p)$, and k_i run over the positive integers $1,2,\dots,\infty$. We can express $K_d(c)$ in terms of the Jacobi function

$$K_d(c) = \int_0^\infty dt e^{-t}\left(\theta_3\left(\frac{it}{c\pi}\right)\right)^d \tag{13}$$

where $\theta_3(z)$ for complex z ($\mathrm{Im}\, z > 0$) is defined by $\theta_3(z) = \Sigma_{k=-\infty}^{\infty} \exp(i\pi k^2 z)$. For the case of more than one extra dimension both the sum of Eq. (12) and the integral of Eq. (13) diverge. To obtain a convergent result the lower limit on the integral must be replaced by a cutoff. The physical origin of the cutoff is a truncation of the sum over the Kaluza-Klein states when their masses exceed the string scale. With the cutoff we obtain the following approximate expressions for $\Delta G_F^{\mathrm{KK}}/G_F^{\mathrm{SM}}$

$$\Delta G_F^{\mathrm{KK}}/G_F^{\mathrm{SM}} \simeq \left[\frac{2\pi^2}{3} + 2\pi \ln\left(\frac{M_{\mathrm{str}}}{M_R}\right)\right]\left(\frac{M_W}{M_R}\right)^2, \quad d=2, \tag{14}$$

$$\Delta G_F^{\mathrm{KK}}/G_F^{\mathrm{SM}} \simeq \left(\frac{d}{d-2}\right)\frac{\pi^{d/2}}{\Gamma(1+d/2)}\left(\frac{M_{\mathrm{str}}}{M_R}\right)^{d-2}\left(\frac{M_W}{M_R}\right)^2, \quad d \geqslant 3. \tag{15}$$

Numerically we find that the approximation of Eq. (15) agrees with the exact result of Eq. (12) to 2–3 % while the approximation of Eq. (14) is good only to about 10% because of the slow convergence of the log function in this case. The quantities M_R and M_{str} are not completely arbitrary but are constrained by the unification of the gauge couplings. Thus in a TeV scale unification the gauge coupling evolution is given by [7,8]

$$\alpha_i(M_Z)^{-1} = \alpha_U^{-1} + \frac{b_i}{2\pi}\ln\left(\frac{M_R}{M_Z}\right) - \frac{b_i^{\mathrm{KK}}}{2\pi}\ln\left(\frac{M_{\mathrm{str}}}{M_R}\right) + \Delta_i. \tag{16}$$

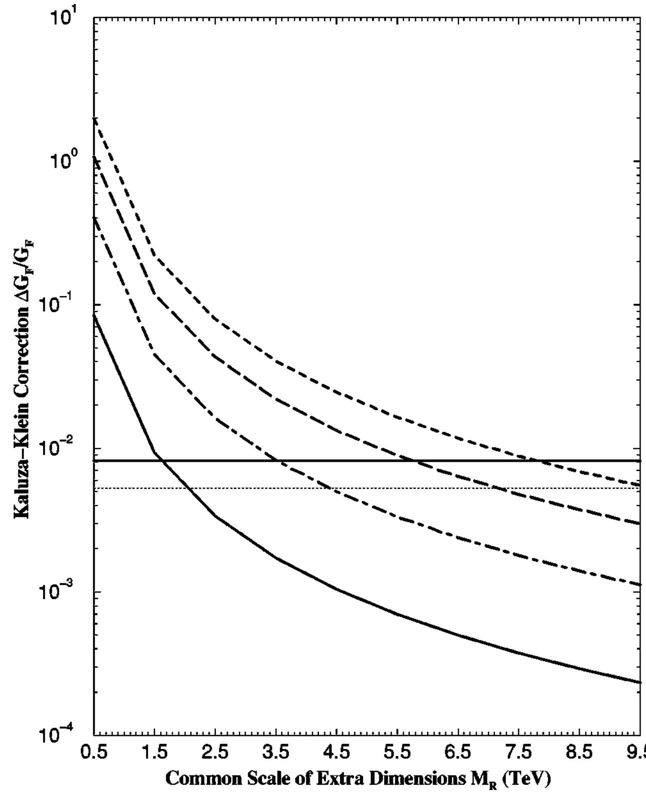

FIG. 1. Plot of the ratio ($\Delta G_F^{\mathrm{KK}}/G_F^{\mathrm{SM}}$) as a function of the common scale of extra dimension. The curves correspond to the case $d=1$ (solid), $d=2$ (dot-dashed), $d=3$ (long dashed), and $d=4$ (dashed). The horizontal solid line corresponds to the 2σ error on $\Delta G_F^{\mathrm{SM}}/G_F^{\mathrm{SM}}$ while the horizontal dotted line is what the 2σ error will be if the error in the W boson measurements decreased by a factor of 2.

Here α_U is the effective grand unification theory (GUT) coupling constant, $b_i = (-3,1,33/5)$ for $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ describe the evolution of the gauge couplings from the scale Q to the scale M_R, $b_i^{\mathrm{KK}} = (-6,-3,3/5)$ are the b_i minus the contribution from the fermion sector which has no Kaluza-Klein excitations, and Δ_i are the corrections arising from the Kaluza-Klein states. The requirement that $\alpha_i(M_Z)$ be compatible with the CERN e^+e^- collider LEP data leads to constraints on M_R and M_{str}. For a given d and M_R the unification of α_1 and α_2 thus fixes the ratio M_{str}/M_R. Thus for a given d and M_R, one uses α_1 and α_2 to compute α_U and M_{str}/M_R. Inserting these values for $i=3$ in Eq. (16) gives $\alpha_3(M_Z)$ within the current error bars. In Ref. [8] an extension to include two additional multiplets F_+ and F_- with $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ quantum numbers $(1,1,+2)$ for F_+ and $(1,1,-2)$ for F_- was proposed with a mass scale M_F in the range $M_{EW} \leqslant M_F \leqslant M_R$. Inclusion of this multiplet improves the agreement of α_3 with experiment but otherwise does not affect our analysis in any substantial way.

The result of our analysis on the lower limit on extra dimensions under the constraints of unification of gauge couplings and under the constraint of Eq. (6) is given in Fig. 1. For the case of one extra dimension the analysis of Fig. 1 shows that the constraint of Eq. (6) puts a lower limit on M_R of 1.6 TeV. This is the same result that we got previously from Eqs. (6) and (7). This is because in this case the unification of the gauge coupling constant constraint gives a high value of $M_{\rm str}/M_R$ so the finite sum on the Kaluza-Klein states simulates to a very good approximation the sum on the infinite tower of Kaluza-Klein states. For the case of 2, 3, and 4 extra dimensions the analysis of Fig. 1 shows that Eq. (6) produces a lower limit on M_R of 3.5 TeV, 5.7 TeV, and 7.8 TeV. In these cases the truncation of the Kaluza-Klein tower is rather severe as one goes to higher values of d, e.g., $M_{\rm str}/M_R \simeq 7.6$ for $d=2$ and this ratio becomes smaller for larger values of d (and further the ratio $M_{\rm str}/M_R$ must be progressively fine tuned more severely as one goes to higher values of d). The specific nature of the cutoff imposed introduces an uncertainty in the accuracy of the predictions which is $O(1\%)$ for $d=1$ and gets larger for larger values of d up to $O(10\%)$ for $d=4$. The largeness of the uncertainty for larger values of d arises due to the progressively smaller number of Kaluza-Klein states that are retained in the truncation procedure under the unification of the gauge coupling constants constraint. As pointed out earlier the analysis is very sensitive to the error in the measurement of the W boson mass. Thus if this error on M_W were to decrease by a factor of 2, which is not an unlikely possibility, then the limit on M_R for $d=1$ will increase to 2.1 TeV, and the limits for $d=2$, $d=3$, and $d=4$ will increase to 4.4 TeV, 7.1 TeV, and 9.5 TeV as can be seen by the intercepts of the dotted line in Fig. 1. These results may be contrasted with the previous limits of ~ 300 GeV from an analysis of contact interactions [7]. Thus none of the Kaluza-Klein excitations of the W or Z boson will become visible at the Tevatron. However, Kaluza-Klein excitations may still be accessible at the CERN Large Hadron Collider (LHC) [16].

One may ask to what extent our limits are dependent on the assumption of an underlying supersymmetry. Since loop corrections to Δr from the Kaluza-Klein states are ignored, which turns out to be a reasonable assumption in view of the largeness of the masses of the Kaluza-Klein states, the essential constraint from supersymmetry arises via the cutoff. For the 5D case this constraint is rather mild but does get severe as one goes to a larger number of extra dimensions. Thus our conclusion is that the 5D limit is essentially model independent but the limits deduced for the 6D–8D cases do require the underlying supersymmetric framework and the results here are thus more model dependent.

Finally we note that the limits on extra dimensions are constrained severely by the errors in the standard model predictions of G_F which are dominated by the error in the W mass measurement and subdominated by the error in the top mass measurement. Thus if future experiments further reduce the errors in these measurements, then one will obtain more stringent bounds on the radius of compactification, or if one sees a deviation from the standard model prediction it might be a smoking gun signal for a TeV scale compactification.

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